Discussion 8 Worksheet

Gradients and directional derivatives

Date: 9/17/2021

MATH 53 Multivariable Calculus

1 Gradients

Compute the gradients of the following functions.

(a) $f(\theta, \phi) = \cos\theta\cos\phi$

(b) $f(x, y) = \arctan(y/x)$

(c) $f(t, x, y) = \frac{1}{\sqrt{4\pi t}}\exp(-(x-y)^2/4t))$. (Express everything as multiples of $f(t, x, y)$.)

2 Directional Derivatives

1. Consider $f(x, y) = e^{-r^4}$ where $r = \sqrt{x^2+y^2}$. Compute its directional derivative at $(0, 0)$ w.r.t. the unit vectors in (polar) directions $\theta = 0, \pi/4, \pi/2$. What about any other angle?

3 Rate of Change

Find the maximum rate of change of f at the given point and direction in which it occurs.

(a) $f(x, y) = 4y\sqrt{x}$ and $(4, 1)$;

(b) $f(x, y) = \sin(xy)$ at $(1, 0)$;

(c) $f(x, y, z) = x/(y+z)$ at $(8, 1, 3)$.

(d) Find all points at which the direction of fastest change of the function $f(x, y) = x^2+y^2-2x-4y$ is $\langle 1, 1\rangle$.

4 True/False

(a) T F Partial derivatives are just a special case of directional derivatives.

(b) T F Consider $f(x, y) = (\cos x + (1+x)\tan y)e^{x^2-1+y}$. Then $\frac{d}{dt}f(t^2, t^3) = 0$ at $t = 0$.

(c) T F Suppose you compute the gradient of $v(s, T) = s/T$, where s has units of miles and T has units of hours. Then the magnitude of ∇v is well-defined.

(d) T F When $\vec{u} \perp \nabla f$ at a point, then $D_{\vec{u}}f = 0$ at that point.

(e) T F If $\vec{u} = \langle a, b\rangle$ is a unit vector and f has continuous second partials, then $D^2_{\vec{u}}f(x, y) = f_{xx}a^2 + 2f_{xy}ab + f_{yy}b^2$ where $D^2_{\vec{u}}f = D_{\vec{u}}[D_{\vec{u}}f(x, y)]$.

Note: These problems are taken from the worksheets for Math 53 in the Spring of 2021 with Prof. Stankova.